I

"Roots of a Complex Number"

M. Abdulla
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1° "Complex Number" in "Rectangular format":

$$\boxed{y = \alpha + j\beta}$$

2° "Complex Number" in "Polar format":

$$y = r e^{j\theta} = \underset{k=0,1,2,\cdots}{r e^{j(\theta \pm 2k\pi)}} = r e^{j(\theta + 2k\pi)}$$

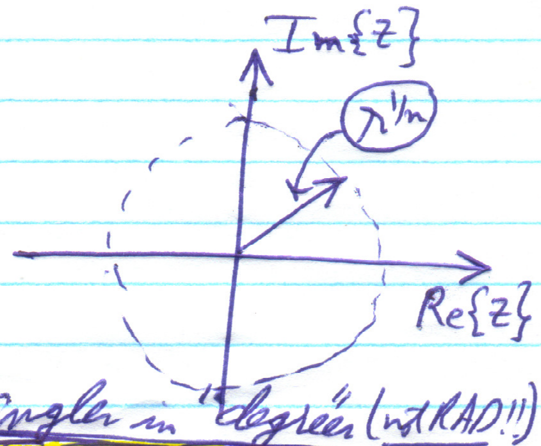
remove "−" because it will produce a redundant result

$$\therefore \boxed{\underset{k=0,1,2,\cdots}{y = r e^{j(\theta + 2k\pi)}}}$$

3° "Roots of a Complex Number":

$$z = \sqrt[n]{y} = (y)^{1/n} = \left(r e^{j(\theta + 2k\pi)}\right)^{1/n} = r^{1/n} e^{j(\theta + 2k\pi)/n}$$

$$\therefore \boxed{\underset{k=0,1,2,\cdots,n-1}{z = r^{1/n} e^{j(\theta + 2k\pi)/n}}}$$



4° How to plot the roots?

$$\left.\begin{aligned} \theta_{rad} = \frac{(\theta + 2k\pi)}{n} &\longrightarrow \theta^\circ \\ \pi &\longrightarrow 180^\circ \end{aligned}\right\} \therefore$$

angles in "degrees" (not RAD!!)

$$\boxed{\underset{k=0,1,2,\cdots,n-1}{\theta^\circ = \frac{180(\theta + 2k\pi)}{n\pi}}}$$

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EX#1: Compute the "6th roots" of "Unity"

Step #1 $\rightarrow$ $y = 1 + j(0) = 1$ (rectangular format)

Step #2 $\rightarrow$ $y = 1\,e^{j(0)} = 1\,e^{j(0 \pm 2k\pi)} = 1\,e^{j(2k\pi)}$

$k = 0, 1, 2, ---$

Step #3 $\rightarrow$ $z = \sqrt[6]{(y)} = (y)^{1/6} = \left(1\,e^{j(2k\pi)}\right)^{1/6}$

$k = 0, 1, 2, 3, 4, 5$

$z = (1)^{1/6}\, e^{j\frac{2k\pi}{6}}$

The roots

$$\boxed{z = e^{j\frac{k\pi}{3}} \quad k = 0, 1, 2, 3, 4, 5}$$

Step #4 $\rightarrow$ $\theta_{rad} = \frac{k\pi}{3} \rightarrow \theta^\circ$, $\pi \rightarrow 180$ $\therefore$ $\theta^\circ = \left(\frac{k\pi}{3}\right)\frac{(180)}{\pi}$

angles in degrees

$$\boxed{\theta^\circ = 60k \quad k = 0, 1, 2, 3, 4, 5}$$

Step #5 $\rightarrow$

$\boxed{z_0} = e^{j(0)} = \boxed{1}$

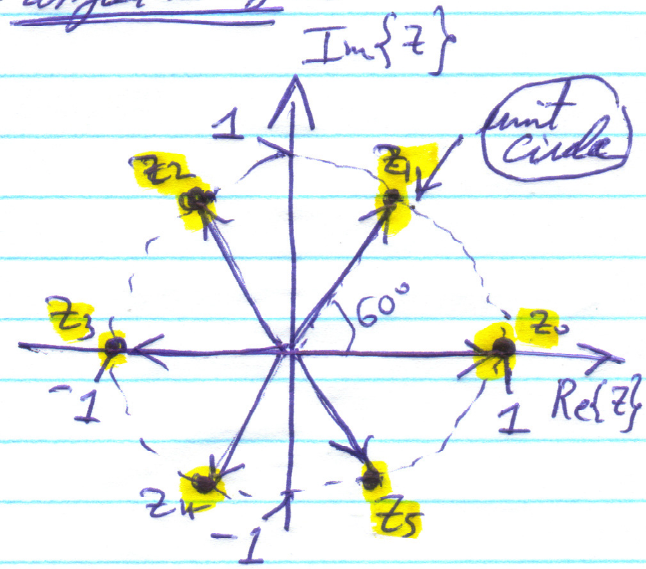
$\boxed{z_1} = e^{j\frac{\pi}{3}} = \boxed{e^{j60^\circ}}$

$\boxed{z_2} = e^{j\frac{2\pi}{3}} = \boxed{e^{j120^\circ}}$

$\boxed{z_3} = e^{j\pi} = \boxed{e^{j180^\circ}}$

$\boxed{z_4} = e^{j\frac{4\pi}{3}} = \boxed{e^{j240^\circ}}$

$\boxed{z_5} = e^{j\frac{5\pi}{3}} = \boxed{e^{j300^\circ}}$



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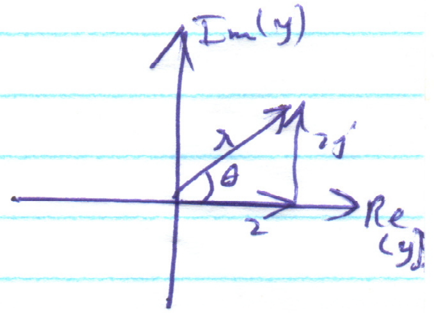
EX#2: Compute the "4th Roots" of "$y = 2 + 2j$"



Step #1: $y = 2 + 2j \longleftarrow$ Rectangular Format

Step #2: $y = \sqrt{(2)^2 + (2)^2}\; e^{j\arctan\left(\frac{2}{2}\right)}$

$$= \sqrt{4+4}\; e^{j\arctan(1)}$$

$$= \sqrt{8}\; e^{j45^\circ}$$

$$= \sqrt{2\times 4}\; e^{j45^\circ} = 2\sqrt{2}\, e^{j45^\circ}$$

$$= 2\sqrt{2}\, e^{j\pi/4} = 2\sqrt{2}\, e^{j(\pi/4 + 2\pi k)}$$

$$= 2\sqrt{2}\, e^{j\left(\frac{\pi}{4} + \frac{8\pi k}{4}\right)} = 2\sqrt{2}\, e^{j\frac{\pi}{4}(1+8k)} \qquad k = 0, 1, 2, \ldots$$

Step #3: $Z = \sqrt[4]{y} = (y)^{1/4} = \left(2\sqrt{2}\, e^{j\frac{\pi(1+8k)}{4}}\right)^{1/4} \qquad k = 0, 1, 2, 3$

$$= \underbrace{\left(2\sqrt{2}\right)^{1/4}}\; e^{j\frac{\pi(1+8k)}{16}} \qquad k = 0, 1, 2, 3$$

$$\left(2^1\, 2^{1/2}\right)^{1/4} = \left(2^{3/2}\right)^{1/4} = 2^{\frac{3}{2}\cdot\frac{1}{4}} = 2^{3/8}$$

Ans: $\boxed{Z = 2^{3/8}\, e^{j\frac{(1+8k)}{16}\pi} \qquad k = 0, 1, 2, 3}$ $\longleftarrow$ The roots

Step #4:

$$\theta_{rad} = \frac{(1+8k)\pi}{16} \longrightarrow \theta^\circ$$

$$\pi \longrightarrow 180^\circ$$

$$\longrightarrow \theta^\circ = \frac{(1+8k)\cancel{\pi}\, 180}{16\, \cancel{\pi}}$$

Angles in degrees $\longrightarrow$ $\boxed{\theta^\circ = 45(1+8K)/4 \quad k = 0, 1, 2, 3}$

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Step #5:

$$\boxed{z_0} = 2^{3/8} e^{j\pi/16} = \boxed{2^{3/8} e^{j\,11.25°}} \checkmark$$

$$\boxed{z_1} = 2^{3/8} e^{j9\pi/16} = \boxed{2^{3/8} e^{j\,101.25°}} \checkmark$$

$$\boxed{z_2} = 2^{3/8} e^{j17\pi/16} = \boxed{2^{3/8} e^{j\,191.25°}} \checkmark \qquad \boxed{2^{3/8} \simeq 1.3}$$

$$\boxed{z_3} = 2^{3/8} e^{j25\pi/16} = \boxed{2^{3/8} e^{j\,281.25°}}$$

We plot the roots