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"De Moivre's Theorem"

M. Abdulla
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$$\boxed{\underbrace{\left(\cos(x)+j\sin(x)\right)^n}_{LHS}=\underbrace{\cos(nx)+j\sin(nx)}_{RHS}}$$

Proof:

$$\overbrace{\left(\underbrace{\cos(x)}_{\triangleq x_0}+j\underbrace{\sin(x)}_{\triangleq y_0}\right)^n}^{LHS}=\left(x_0+jy_0\right)^n=\left(\sqrt{x_0^2+y_0^2}\;e^{j\arctan\left(\frac{y_0}{x_0}\right)}\right)^n$$

$$=\left(\sqrt{\cos^2(x)+\sin^2(x)}\;e^{j\arctan\left(\frac{\sin(x)}{\cos(x)}\right)}\right)^n$$

$$=\left(e^{j\arctan\left(\tan(x)\right)}\right)^n=\left(e^{jx}\right)^n=e^{jnx}$$

$$=\underbrace{\cos(nx)+j\sin(nx)}_{RHS}\qquad \text{Q.E.D.}\ \blacksquare$$

EX: $\boxed{z=\frac{1}{2}+j\frac{\sqrt{3}}{2}}$ find z^5

$$z=\frac{1}{2}+j\frac{\sqrt{3}}{2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\;e^{j\arctan\left(\frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}}\right)}$$

$$=\sqrt{\frac{1}{4}+\frac{3}{4}}\;e^{j\arctan\left(\sqrt{3}\right)}=e^{j\arctan\left(\sqrt{3}\right)}$$

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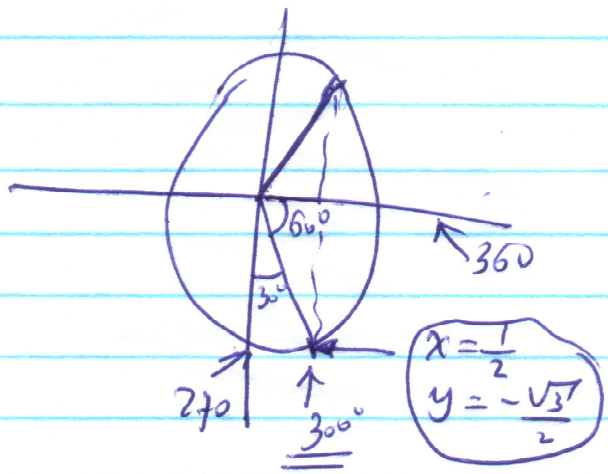
$$\boxed{Z} = e^{j\arctan(\sqrt{3})} = \boxed{e^{j60^\circ}}$$

$$Z^5 = (e^{j60^\circ})^5 = e^{j300^\circ} = \cos(300) + j\sin(300)$$

$$\boxed{Z^5 = \frac{1}{2} - j\frac{\sqrt{3}}{2}}$$

Answer!



NB: if you forget the explicit relationship of "De Moivre's Theorem", then No Worries!

Simply change a "Rectangular format" to "polar" to solve an "exponent type question".

$\boxed{Z = \alpha + j\beta} \rightarrow \boxed{Z = r e^{j\theta}} \rightarrow \boxed{Z^n = \alpha_0 + j\beta_0}$

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