

Mean Value Theorem (M.V.T.)

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What is "MVT"? ["MVT not MTV!!!"] 😊

$$f'(c) = \frac{f(b) - f(a)}{(b-a)}$$

Warning!!
a & b are EXCLUDED
Not included

Get: "c" $\rightarrow \{c \in \mathbb{R} \mid a < c < b\}$

it is assumed that: 1) $f(x): [a, b] \rightarrow \mathbb{R}$ is "continuous" and "differentiable"
2) $a < b$ $(a, b) \in \mathbb{R}^2$
3) $f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} \leftarrow$ exists
over $x \in (a, b)$

here the function is inclusive

closed

Open

We will do some examples \rightarrow Next Page!

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EX #1: $f(x) = x^3 + 3x^2 \quad x \in [-5, 1]$

Apply AMVT

1° $f'(x) = 3x^2 + 6x = 3x(x+2) \quad \begin{matrix} a = -5 \\ b = 1 \end{matrix}$

2° $f'(c) = 3c(c+2) = \frac{f(1) - f(-5)}{(1+5)}$

$3c(c+2) = \frac{1}{6} [4 + 50] = \frac{54}{6} = 9$

$f(1) = 1 + 3 = 4$
 $f(-5) = (-5)^3 + 3(-5)^2 = -125 + 75 = -50$

$c(c+2) = 3$

$c^2 + 2c - 3 = 0$

$\begin{matrix} a = 1 \\ b = 2 \\ c = -3 \end{matrix}$

$c = \frac{-2 \pm \sqrt{4 + 4 \cdot 1 \cdot 3}}{2 \cdot 1} = \frac{-2 \pm 4}{2} = -1 \pm 2 \rightarrow \begin{matrix} 1 \\ -3 \end{matrix}$

$c = 1$ is Not in $(-5, 1)$

$(c-1)(c+3) = 0 \implies c = 1 \leftarrow$ Not in $(-5, 1)$

Answer $c = -3$

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Ex #2:

$$f(x) = \frac{1}{x^2} \quad x \in [-1, 1]$$

Apply MVT

$$\underline{1^o} \quad \boxed{f'(x)} = (x^{-2})' = -2x^{-3} = \boxed{\frac{-2}{x^3}}$$

$$\boxed{f'(c) = \frac{-2}{c^3}}$$

$$\underline{2^o} \quad \left. \begin{array}{l} a = -1 \\ b = 1 \end{array} \right\} \rightarrow \begin{array}{l} f(a) = f(-1) = \frac{1}{(-1)^2} = 1 \\ f(b) = f(1) = \frac{1}{(1)^2} = 1 \end{array}$$

$$\underline{3^o} \quad f'(c) = \frac{-2}{c^3} = \frac{(1-1)}{(1+1)} = \frac{0}{2} = 0$$

$$\boxed{\frac{-2}{c^3} = 0}$$

No solution for "c"

Answer!

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EX#3:

$$f(x) = 3x^2 + 2x + 5 \quad x \in [-1, 1]$$

Apply MVT

closed

1^o $f'(x) = 6x + 2 = 2(3x + 1)$

$$f'(c) = 2(3c + 1)$$

2^o $a = -1$
 $b = 1$

$$f(a) = f(-1) = 3 - 2 + 5 = 6$$

$$f(b) = f(1) = 3 + 2 + 5 = 10$$

3^o $f'(c) = \frac{f(b) - f(a)}{b - a}$

$$2(3c + 1) = \frac{10 - 6}{1 - (-1)} = \frac{4}{2} = 2$$

$$2(3c + 1) = 2$$

$$3c + 1 = 1$$

$$3c = 0$$

$$c = \frac{0}{3} = 0$$



Finally done!

$$c = 0$$

Answer!

Valid because $c \in (-1, 1)$

open

The end.