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"Descartes' Rule of Signs" (D.R.S.)

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- D.R.S. will "NOT" tell us "where" the zeros are located.
- D.R.S. tells us "How Many Roots" we can "Expect" of a "Polynomial".
- Reminders:

What is a Polynomial?

1st order Poly. → $P_1(x) = a_0 + a_1 x$

2nd order Poly. → $P_2(x) = a_0 + a_1 x + a_2 x^2$

3rd order Poly. → $P_3(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

⋮

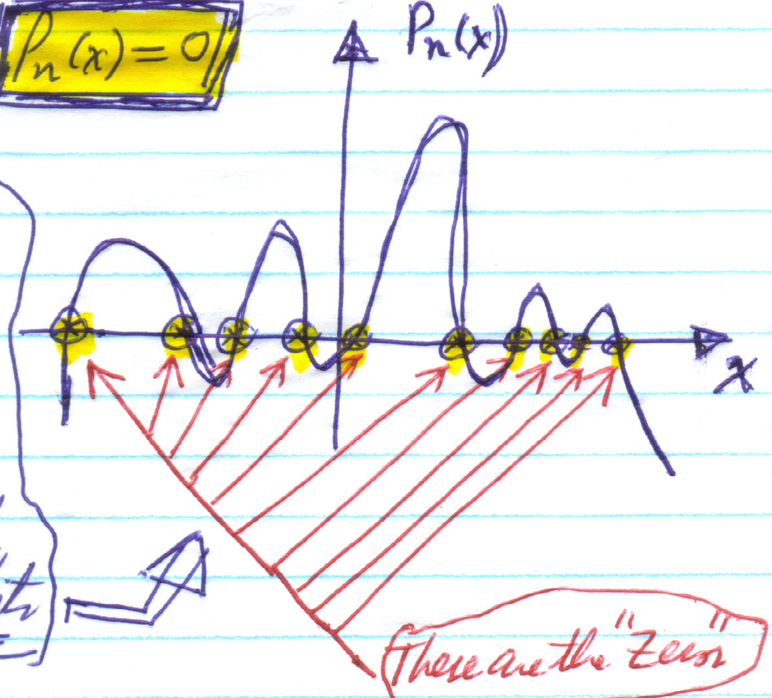
nth order Poly. → $P_n(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$

$$\boxed{P_n(x) = \sum_{i=0}^{n} a_i x^i} \quad n \in \mathbb{N}^*$$

What are Zeros?

To get the "Zeros" → set $\boxed{P_n(x) = 0}$

The value of "x" that produces this are called "Zeros". In other words we want to find all "x-intercepts"



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EX#1 Use DR's to find the potential amount of Zeros for this Polynomial

$$f(x) = x^5 - x^4 + 3x^3 + 9x^2 - x + 5$$

Step #1 $\Rightarrow$ $f(x) = +x^5 - x^4 + 3x^3 + 9x^2 - x + 5$

We identify the "sign change" of coefficient

Step #2 $\Rightarrow$ $f(-x) = (-x)^5 - (-x)^4 + 3(-x)^3 + 9(-x)^2 - (-x) + 5$

$$= -x^5 - x^4 - 3x^3 + 9x^2 + x + 5$$

Step #3 $\Rightarrow$

+ve Zeros: 4 or 2 or 0

-ve Zeros: 1

Roots in "$\mathbb{R}$"

	-ve Zeros
+ve Zeros	1
0	1
2	3
4	5

Possible amount of Zeros

Answer

Possible amount of Zeros

$\boxed{1 \text{ or } 3 \text{ or } 5}$

$\boxed{3}$

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Ex#2 Apply the concept of D.R.S. to

$$f(x) = 4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1$$

Step#1 $\Rightarrow f(x) = +4x^7 + 3x^6 + x^5 + 2x^4 - x^3 + 9x^2 + x + 1$

Step#2 $\Rightarrow f(-x) = 4(-x)^7 + 3(-x)^6 + (-x)^5 + 2(-x)^4 - (-x)^3 + 9(-x)^2 + (-x) + 1$

$$= -4x^7 + 3x^6 - x^5 + 2x^4 + x^3 + 9x^2 - x + 1$$

Step#3 $\Rightarrow$ Possible Roots in $\mathbb{R}$

- +ve: 2 or 0
- −ve: 5 or 3 or 1

	−ve Zeros		
+ve Zeros	1	3	5
0	1	3	5
2	3	5	7

Possible amt of Zeros

Possible amt of Zeros

$\boxed{1 \text{ or } 3 \text{ or } 5 \text{ or } 7}$

answer

4

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EX#3: Apply DRS to:

$$f(x) = x^5 + x^4 + 4x^3 + 3x^2 + x + 1$$

Step#1: $f(x) = + x^5 + x^4 + 4x^3 + 3x^2 + x + 1$

No Transitoin! (sign)
0

Step#2:

$$f(-x) = (-x)^5 + (-x)^4 + 4(-x)^3 + 3(-x)^2 + (-x) + 1$$

$$= - x^5 + x^4 - 4x^3 + 3x^2 - x + 1$$

Step#3:

Possible Roots in $\mathbb{R}$ → $+ve$: 0

Possible Roots in $\mathbb{R}$ → $-ve$: 5 & 3 & 1

Possible kind of Zeros.

$\boxed{1 \ \& \ 3 \ \& \ 5}$ ← Answer

$\boxed{5}$ Apply D.R.S. to:

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$$f(x) = 2x^4 - x^3 + 4x^2 - 5x + 3$$

Step #1: $f(x) = +2x^4 - x^3 + 4x^2 - 5x + 3$

Step #2:

$$f(-x) = 2(-x)^4 - (-x)^3 + 4(-x)^2 - 5(-x) + 3$$

$$= +2x^4 + x^3 + 4x^2 + 5x + 3$$

No sign transitions.

Step #3:

Possible amount of Zeros in $\mathbb{R}$ → +ve: 4 or 2 or 0

→ −ve: 0

Possible amount of Zeros

$\boxed{0 \text{ or } 2 \text{ or } 4}$ ← answer

$\boxed{6}$ Apply D.R.S! to:

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$$f(x) = x^4 - x^3 - 9x^2 - 3x - 36$$

Step #1: $f(x) = +x^4 - x^3 - 9x^2 - 3x - 36$

Step #2: $f(-x) = (-x)^4 - (-x)^3 - 9(-x)^2 - 3(-x) - 36$

$$= +x^4 + x^3 - 9x^2 + 3x - 36$$

Step #3:

Possible count of zeros in $\mathbb{R}$

+ve: 1

-ve: 3 or 1

Cross correlation

	-ve zeros	
+ve zeros	1	3
1	2	4

$\boxed{2 \text{ or } 4}$ ← Possible count of zeros ← answer!

The End