Plotting a Rational Function

M. Abdulla
Oct. 23. 2011

EX#1:

$$\boxed{f(x) = \frac{(2x+1)}{x(x-1)(x+2)}} \Leftarrow \text{Plot this!}$$

Step#1:

→ Find the Vertical Asymptotes (V.A.)

$$x(x-1)(x+2) = 0 \Rightarrow \boxed{\begin{matrix} x = 0 \\ x = 1 \\ x = -2 \end{matrix}}$$

3 V.A.'s

Step#2

→ Find Horizontal Asymptote (H.A.) or Oblique Asymptote (O.A.)

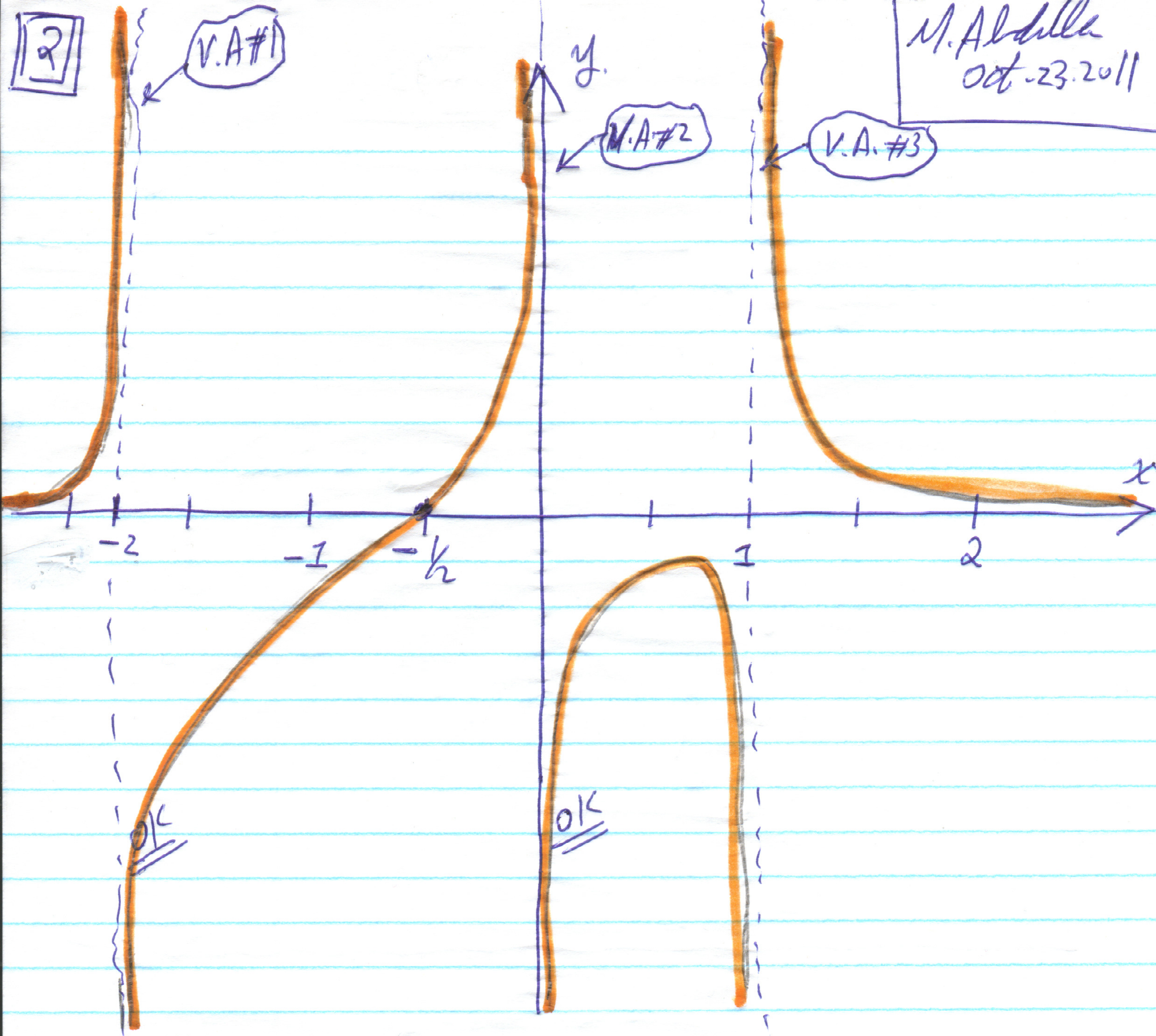
$$f(x) = \frac{n(x) \leftarrow \text{order} \Rightarrow 1}{d(x) \leftarrow \text{order} \Rightarrow 3} \quad \Big\} \quad 1 < 3 \therefore \text{No H.A.}, \ \text{No O.A.}$$

Step#3

→ Perform various "limits" to figure out how to plot this Rational function.

$\boxed{2}$

M. Abdulla
Oct-23-2011



1° $$\boxed{\lim_{x\to-\infty} f(x)} = \lim_{x\to-\infty} \frac{(2x+1)}{x(x-1)(x+2)} = \lim_{x\to-\infty} \frac{(2x+1)/x}{x(x-1)(x+2)/x}$$

$$= \lim_{x\to-\infty} \frac{(2+1/x)}{(x-1)(x+2)} = \frac{(2+0)}{(-\infty)(-\infty)} = \frac{2}{\infty} = \boxed{0}$$

2° $$\boxed{\lim_{x\to\infty} f(x)} = \lim_{x\to\infty} \frac{(2+1/x)}{(x-1)(x+2)} = \frac{(2+0)}{(\infty)(\infty)} = \frac{2}{\infty} = \boxed{0}$$

$\boxed{3}$

M. Abdulla
oct. 23. 2011

3° $\boxed{\lim_{x \to -2} f(x)} = \lim_{x \to -2} \frac{(2x+1)}{x(x-1)(x+2)} = \lim_{x \to 2} \frac{((2)(-2)+1)}{(-2)(-2-1)(x+2)}$

$= \lim_{x \to -2} \frac{\cancel{(-3)}}{(-2)\cancel{(-3)}(x+2)} = \boxed{\left(\frac{-1}{2}\right) \lim_{x \to -2} \frac{1}{(x+2)}}$

→ **Case #1:** $\boxed{\lim_{x \to -2^-} f(x)} = \left(-\frac{1}{2}\right) \frac{(-1)}{0} = \frac{1}{2}\infty = \boxed{\infty}$ ✓

→ **Case #2:** $\boxed{\lim_{x \to -2^+} f(x)} = \left(-\frac{1}{2}\right) \frac{1}{0} = \boxed{-\infty}$ ✓

4° $\boxed{\lim_{x \to 0} f(x)} = \lim_{x \to 0} \frac{(2x+1)}{x(x-1)(x+2)} = \lim_{x \to 0} \frac{(1)}{x(-1)(2)}$

$= \boxed{\left(\frac{-1}{2}\right) \lim_{x \to 0} \frac{1}{x}}$

→ **Case #1:** $\boxed{\lim_{x \to 0^-} f(x)} = \left(-\frac{1}{2}\right)(-\infty) = \boxed{\infty}$ ✓

→ **Case #2:** $\boxed{\lim_{x \to 0^+} f(x)} = \left(\frac{-1}{2}\right)(\infty) = \boxed{-\infty}$ ✓

M. Abdulla
Oct-23-2011

5° $\boxed{\lim_{x\to 1} f(x)} = \lim_{x\to 1} \frac{(2x+1)}{x(x-1)(x+2)}$

$= \lim_{x\to 1} \frac{\cancel{(3)}}{(1)(x-1)\cancel{(3)}} = \boxed{\lim_{x\to 1} \frac{1}{(x-1)}}$

→ Case #1: $\boxed{\lim_{x\to 1^-} f(x)} = \frac{-1}{0} = \boxed{-\infty}$ ✓

→ Case #2: $\boxed{\lim_{x\to 1^+} f(x)} = \frac{1}{0} = \boxed{\infty}$ ✓

Step #4

→ find $f(x) = 0$ "$x = ?$" ← the Zero.

$f(x) = \frac{(2x+1)}{x(x-1)(x+2)} = 0$

& $(2x+1) = 0$

→ $2x = -1$

→ $\boxed{x_0 = \frac{-1}{2}}$ ← Zero. ✓

Now, we have all the components necessary to plot the function check P. 2

$\boxed{5}$

M. Abdulla
Oct-23-2011

Ex#2: $\boxed{f(x) = \dfrac{(3x^2+2)}{(x-3)}}$ ⟸ Plot this!

Step#1: Find H.A. $\quad x-3=0 \quad \therefore \boxed{x=3}$ H.A.

Step#2 Find V.A. or O.A.

$$f(x) = \frac{n(x) \leftarrow \text{order}=2}{d(x) \leftarrow \text{order}=1} \Bigg\} \; 2 \geq 1 \; \therefore \text{O.A.}$$

Perform long-division:

$$\begin{array}{r|l} (3x^2+2) & x-3 \\ \hline -(3x^2-9x) & 3x+9 \\ \cancel{3x^2}+2-\cancel{3x^2}+9x & \\ =(9x+2) & \\ -(9x-27) & \\ \cancel{9x}+2-\cancel{9x}+27 & \\ =\boxed{29} & \end{array}$$

O.A.

$$\therefore \boxed{f(x) = \frac{(3x^2+2)}{(x-3)} = (3x+9) + \frac{29}{(x-3)}} \quad (*)$$

$\boxed{6}$

M. Abdulla
Oct-23-2011

Let's check if (*) is indeed correct:

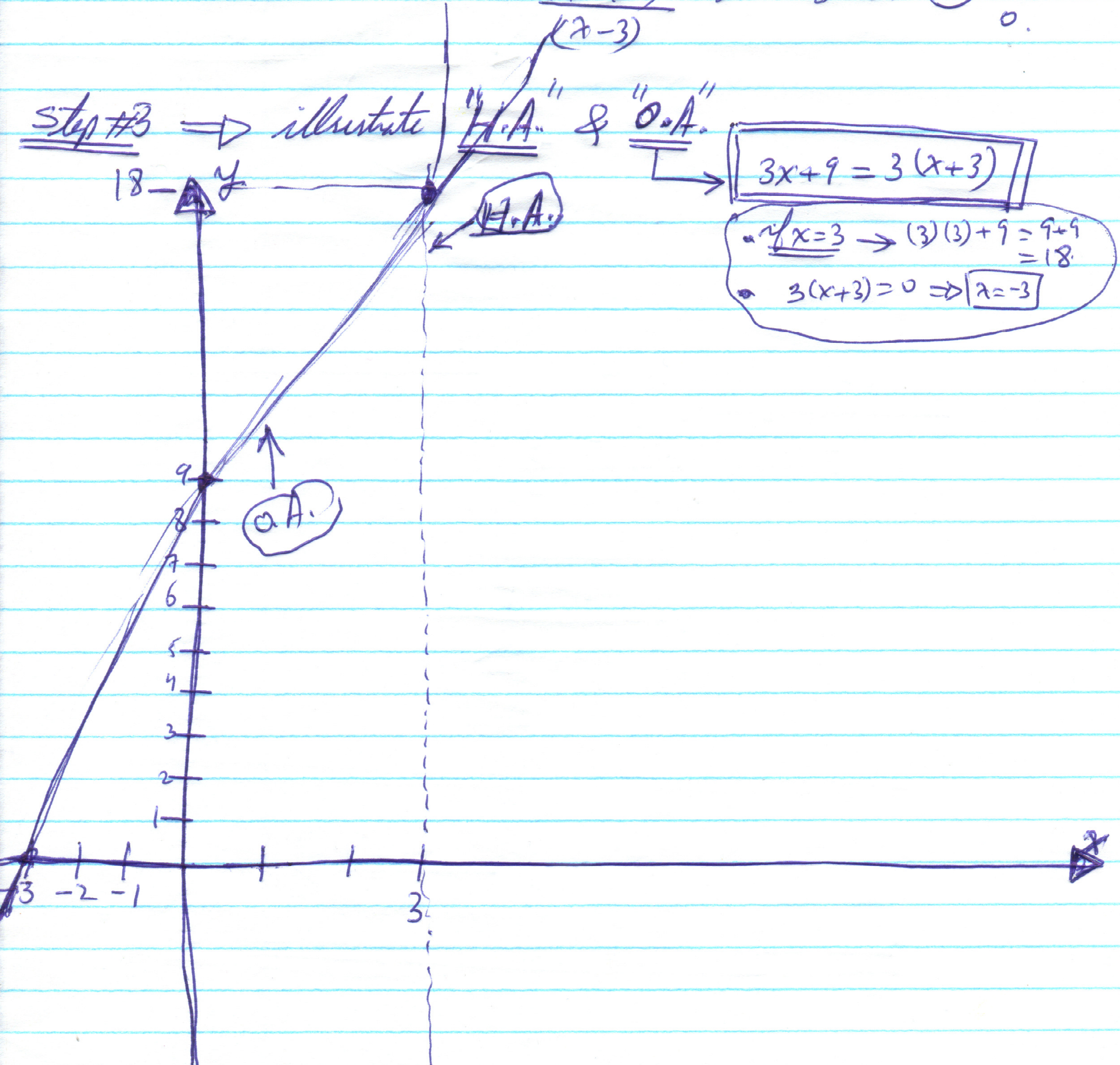
$$\frac{(3x+9)(x-3)+29}{(x-3)} = \frac{3x^2 - \cancel{9x} + \cancel{9x} - 27 + 29}{(x-3)}$$

$$= \frac{(3x^2+2)}{(x-3)} \quad \checkmark \text{ correct} \ ☺ \ !$$

Step #3 $\Rightarrow$ illustrate "H.A." & "O.A."

$\boxed{3x+9 = 3(x+3)}$

- if $x=3 \rightarrow (3)(3)+9 = 9+9 = 18$
- $3(x+3)=0 \Rightarrow \boxed{x=-3}$



M. Abdulla
Oct. 23. 2011

step #4 take various related limits

1° $\boxed{\lim_{x \to -\infty} f(x)} = \lim_{x \to -\infty} \frac{(3x^2+2)}{(x-3)} = \lim_{x \to -\infty} \frac{6x}{1} = (6)(-\infty) = \boxed{-\infty}$

2° $\boxed{\lim_{x \to \infty} f(x)} = \lim_{x \to \infty} 6x = (6)(\infty) = \boxed{\infty}$

3° $\boxed{\lim_{x \to 3} f(x)} = \lim_{x \to 3} \frac{(3x^2+2)}{(x-3)} = \lim_{x \to 3} \frac{((3)(9)+2)}{(x-3)}$

$= \boxed{(29) \lim_{x \to 3} \frac{1}{(x-3)}}$

→ Case #1: $\boxed{\lim_{x \to 3^-} f(x)} = (29)\frac{-1}{0} = \frac{-29}{0} = \boxed{-\infty}$

→ Case #2: $\boxed{\lim_{x \to 3^+} f(x)} = (29)\frac{(+1)}{(0)} = \boxed{\infty}$

---

step #5: $f(x) = 0 \rightarrow$ get "x"

$\frac{(3x^2+2)}{(x-3)} = 0 \quad \therefore \quad 3x^2+2=0$

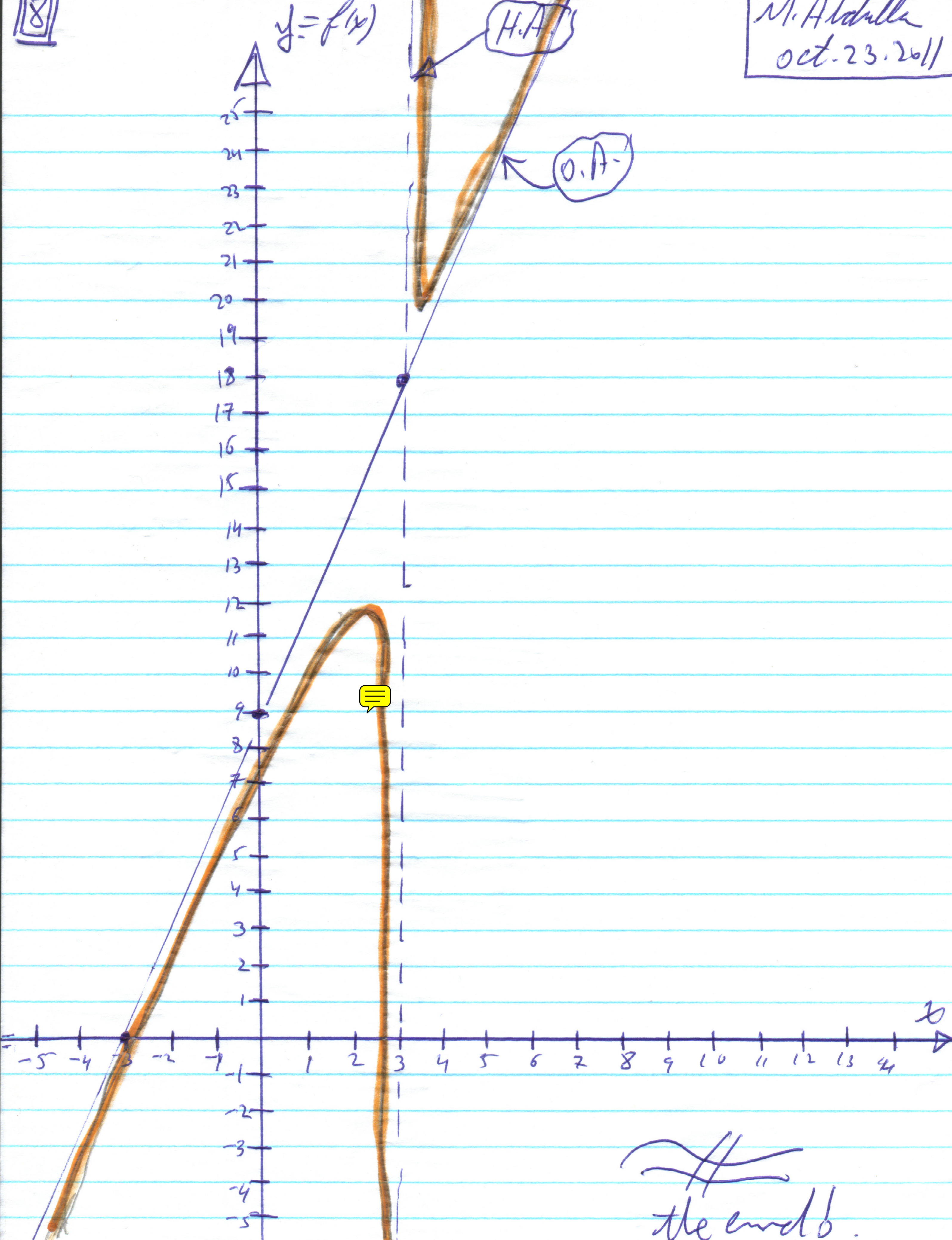
$3x^2 = -2$

$x^2 = \frac{-2}{3}$

Not in "$\mathbb{R}$" $\longrightarrow x = \pm\sqrt{\frac{-2}{3}} = \pm\sqrt{\frac{2}{3}}\, i$

$\boxed{8}$

M. Aldulla
oct. 23. 2011



the end!