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Multiple LTI Systems

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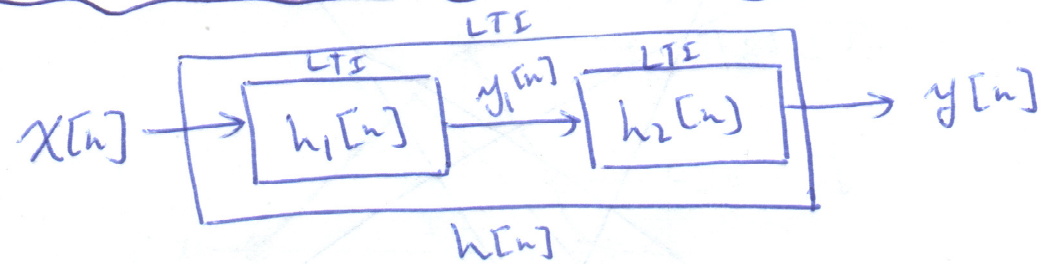
Question #1: → cascade of 2 LTI System

→ $H_1(e^{jw}) = \dfrac{(2 - e^{-jw})}{(1 + \frac{1}{2} e^{-jw})}$

$H_2(e^{jw}) = \dfrac{1}{(1 - \frac{1}{2} e^{-jw} + \frac{1}{4} e^{-j2w})}$

→ Find the "difference equation" for the entire system.

1°



Step #1: $y_1[n] = x[n] * h_1[n] \overset{F.T.}{\Longleftrightarrow} Y_1(e^{jw}) = X(e^{jw}) \cdot H_1(e^{jw})$

Step #2: $y[n] = y_1[n] * h_2[n] \overset{F.T.}{\Longleftrightarrow} Y(e^{jw}) = Y_1(e^{jw}) \cdot H_2(e^{jw})$

$\therefore \; Y(e^{jw}) = X(e^{jw}) \cdot H_1(e^{jw}) \cdot H_2(e^{jw})$

Frequency Domain — Time Domain

$$\boxed{H(e^{jw}) \overset{\Delta}{=} \frac{Y(e^{jw})}{X(e^{jw})} = H_1(e^{jw}) \cdot H_2(e^{jw})} \overset{(FT)^{-1}}{\Longleftrightarrow} \boxed{h[n] = h_1[n] * h_2[n]}$$

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2°

$$H(e^{jw}) = H_1(e^{jw}) \cdot H_2(e^{jw})$$

$$= \frac{(2 - e^{-jw})}{(1 + \frac{1}{2} e^{-jw})} \cdot \frac{1}{(1 - \frac{1}{2} e^{-jw} + \frac{1}{4} e^{-j2w})}$$

$$= \frac{(2 - e^{-jw})}{(1 \cancel{- \frac{1}{2} e^{-jw}} \cancel{+ \frac{1}{4} e^{-j2w}} \cancel{+ \frac{1}{2} e^{-jw}} \cancel{- \frac{1}{4} e^{-j2w}} + \frac{1}{8} e^{-j3w})}$$

$$\frac{Y(e^{jw})}{X(e^{jw})} = \frac{(2 - e^{-jw})}{(1 + \frac{1}{8} e^{-j3w})}$$

$$Y(e^{jw}) \cdot (1 + \frac{1}{8} e^{-j3w}) = X(e^{jw}) \cdot (2 - e^{-jw})$$

$$\boxed{Y(e^{jw}) + \frac{1}{8} e^{-j3w} Y(e^{jw}) = 2X(e^{jw}) - e^{-jw} X(e^{jw})}$$

$(F.T.)^{-1}$

$$\boxed{y[n] + \frac{1}{8} y[n-3] = 2x[n] - x[n-1]}$$

difference Equation for entire cascading system

Answer!

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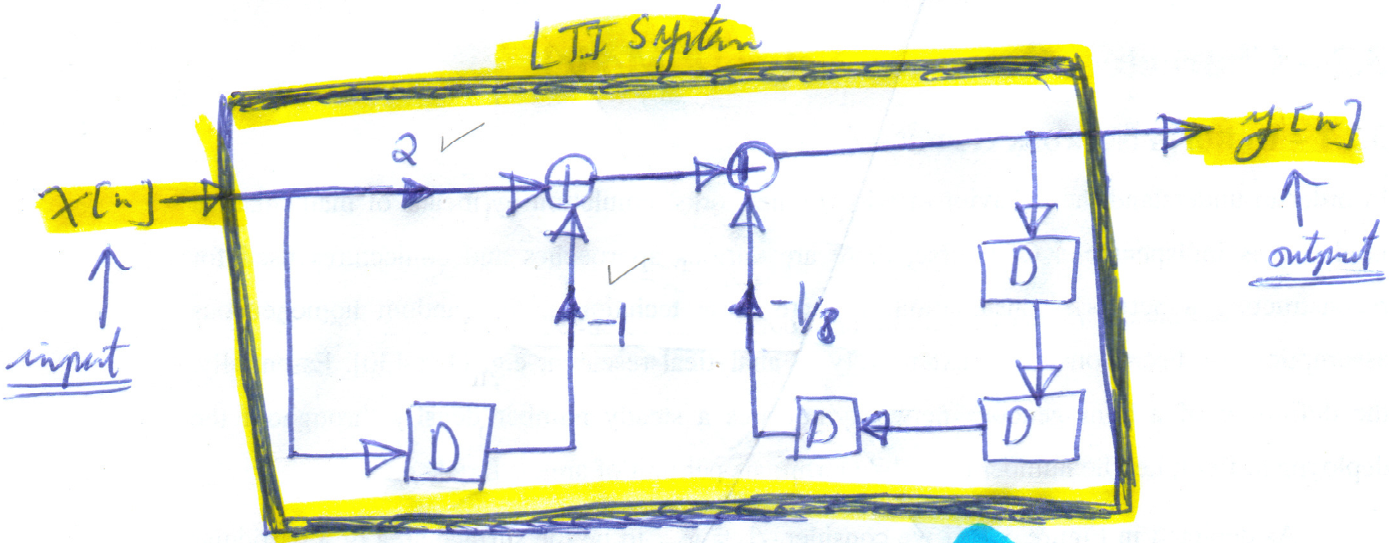
What is the Block Diagram of the above System?

↳ isolate for "$y[n]$"

$$\boxed{y[n] = 2x[n] - x[n-1] - \frac{1}{8}y[n-3]}$$



System

Voila!

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Question #2:

→ 2 LTI systems are connected in parallel

→ The overall Frequency Response is:

$$H(e^{jw}) = \frac{(-12+5e^{-jw})}{(12-7e^{-jw}+e^{-j2w})}$$

→ $h_1[n] = (1/3)^n u[n]$

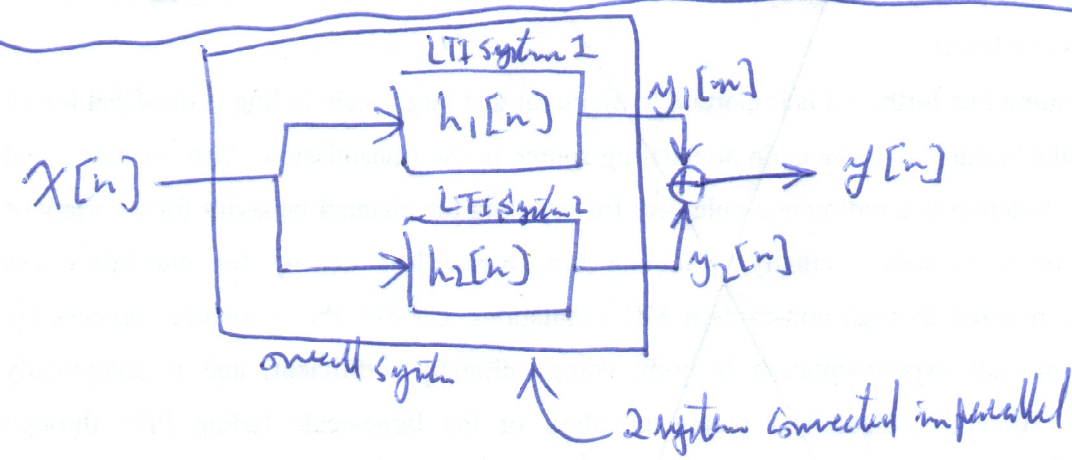
(A) Find "$h_2[n]$" and "$h[n]$"

(B) Find the Difference Equation

(C) Draw the Block Diagram of the System

1°

(A)



2°

$y_1[n] = x[n] * h_1[n]$

$y_2[n] = x[n] * h_2[n]$

$y[n] = y_1[n] + y_2[n]$

$$\boxed{y[n] = (x[n] * h_1[n]) + (x[n] * h_2[n])}$$

F.T.

$$Y(e^{jw}) = X(e^{jw}) \cdot H_1(e^{jw}) + X(e^{jw}) \cdot H_2(e^{jw})$$

$$= X(e^{jw}) [H_1(e^{jw}) + H_2(e^{jw})]$$

$$\therefore \boxed{\frac{Y(e^{jw})}{X(e^{jw})} = H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})}$$

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3°
$$\boxed{H(e^{jw}) = \frac{(-12+5e^{-jw})}{(12-7e^{-jw}+e^{-j2w})} = H_1(e^{jw}) + H_2(e^{jw})}$$

4°
$$h_1[n] = \left(\tfrac{1}{3}\right)^n u[n] \xleftrightarrow{F.T.} \boxed{H_1(e^{jw}) = \frac{1}{(1-\frac{1}{3}e^{-jw})}}$$

$\left|\tfrac{1}{3}\right| < 1$ Yes!

5° What is $H_2(e^{jw})$?

$$\boxed{H_2(e^{jw})} = H(e^{jw}) - H_1(e^{jw})$$
$$= \boxed{\frac{(-12+5e^{-jw})}{(12-7e^{-jw}+e^{-j2w})} - \frac{1}{(1-\frac{1}{3}e^{-jw})}}$$

Can we factor?

6° $\boxed{z = e^{-jw}}$

$z^2 - 7z + 12 = 0$ $\quad a=1$, $b=-7$, $c=12$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{+7 \pm \sqrt{49-(4)(1)(12)}}{(2)(1)} = \frac{7 \pm \sqrt{1}}{2} = \frac{7\pm 1}{2} = \begin{cases} 4 \\ 3 \end{cases}$$

$$\therefore (z-3)(z-4) = (e^{-jw}-3)(e^{-jw}-4)$$
$$= (3-e^{-jw})(4-e^{-jw})$$
$$= 3\left(1-\tfrac{1}{3}e^{-jw}\right)4\left(1-\tfrac{1}{4}e^{-jw}\right)$$
$$= \boxed{12\left(1-\tfrac{1}{3}e^{-jw}\right)\left(1-\tfrac{1}{4}e^{-jw}\right)}$$

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7°

$$\therefore H_2(e^{jw}) = \frac{(-12+5e^{-jw})}{12(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})} - \frac{1}{(1-\frac{1}{3}e^{-jw})}$$

$$= \frac{(-12+5e^{-jw})}{12(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})} - \frac{12(1-\frac{1}{4}e^{-jw})}{12(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})}$$

$$= \frac{-\overset{\checkmark}{12}+5e^{-jw}-\overset{\checkmark}{12}+3e^{-jw}}{12(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})}$$

$$= \frac{-24+8e^{-jw}}{12(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})}$$

$$= \frac{(-2+2/3\, e^{-jw})}{(1-\frac{1}{3}e^{-jw})(1-\frac{1}{4}e^{-jw})}$$

$$= \frac{(-2)\cancel{(1-1/3\, e^{-jw})}}{\cancel{(1-1/3\, e^{-jw})}(1-1/4\, e^{-jw})} = \frac{(-2)}{(1-1/4\, e^{-jw})}$$

$$\therefore \boxed{H_2(e^{jw}) = \frac{-2}{(1-1/4\, e^{-jw})}}$$

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8°
$$\boxed{h_2[n]} = (1/4)^n u[n] \cdot (-2)$$

$$= (-1)\frac{(2)^1}{(2^2)^n} u[n]$$

$$= (-1)\frac{2^1}{2^{2n}} u[n]$$

$$= (-1)\frac{1}{2^{2n-1}} u[n] \triangleq \boxed{(-1)\ 2^{(1-2n)} u[n]}$$

$$\therefore \boxed{h_2[n] = -2^{(1-2n)} u[n]}$$

answer 4

(A)

9° from 8°
$$H(e^{jw}) = H_1(e^{jw}) + H_2(e^{jw})$$

$$\xrightarrow{(FT)^{-1}} h[n] = h_1[n] + h_2[n]$$

$$= 3^{-n} u[n] - 2^{(1-2n)} u[n]$$

$$\triangleq \boxed{h[n] = \left[3^{-n} - 2^{(1-2n)}\right] u[n]}$$

impulse Response of the overall system!

(B) What is the "Difference Equation"?

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{(-12 + 5e^{-jw})}{(12 - 7e^{-jw} + e^{-j2w})}$$

$$Y(e^{jw})\left[12 - 7e^{-jw} + e^{-j2w}\right] = X(e^{jw})\left[-12 + 5e^{-jw}\right]$$

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$$12Y(e^{jw}) - 7e^{-jw}Y(e^{jw}) + e^{-j2w}Y(e^{jw})$$
$$= -12X(e^{jw}) + 5e^{-jw}X(e^{jw})$$

$$\boxed{12y[n] - 7y[n-1] + y[n-2] = -12x[n] + 5x[n-1]}$$

answer to (B)

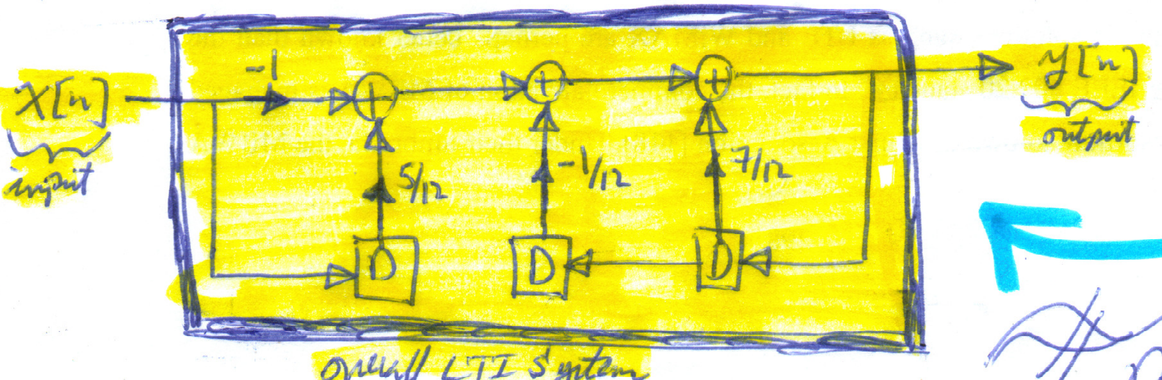
(C) Block Diagram:

1° isolate for "y[n]":

$$\frac{12y[n]}{12} = -\frac{12x[n]}{12} + \frac{5x[n-1]}{12} + \frac{7y[n-1]}{12} - \frac{y[n-2]}{12}$$

so:

$$\boxed{y[n] = -x[n] + (5/12)x[n-1] + (7/12)y[n-1] - \left(\frac{1}{12}\right)y[n-2]}$$



answer (C)